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Pion and photon induced reactions on the nucleon in a unitary model

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Abstract

We present a relativistic calculation of pion scattering, pion photoproduction and Compton scattering on the nucleon in the energy region of the Δ-resonance (upto 450 MeV photon lab energy), in a unified framework which obeys the unitarity constraint. It is found that the recent data on the cross section for nucleon Compton scattering determine accurately the parameters of the electromagnetic nucleon–Δ coupling. The pion-photoproduction partial-wave amplitudes, calculated with parameters fitted to the pion-nucleon and Compton scattering, agree well with the recent Arndt analysis.

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1. Introduction

It is well known that in Compton scattering from the nucleon, by using arguments based on unitarity and causality [1], strong constraints can be put on the cross section. These are usually formulated in terms of the fixed-t dispersion relations [2] expressing the real part of the six Hearn-Leader amplitudes [3] through the imaginary part. The latter is directly related by the optical theorem to the pion-photoproduction cross section. In such an approach the relatively small Born terms are added to account for the low-energy behaviour of the full amplitude. The slow convergence of the dispersion integrals requires several subtraction functions which are related to the t-channel singularities, especially to the $\pi\pi$ exchange [4]. A lower unitary bound on the Compton cross section can be obtained by setting the real parts of the amplitude equal to zero [2] or to the Born contribution [5]. A disadvantage of the dispersion approach is that the link with the decay properties of the nucleon resonances is getting obscured. In particular it is difficult to extract the contribution of the Δ-resonance to Compton scattering and parameters of the decay $\Delta \to N + \gamma$.

In order to express more clearly the contribution of the Δ-resonance, a relativistic tree-level calculation was performed [6] for nucleon Compton scattering. In this calculation the Δ-resonance was included via the Rarita-Schwinger propagator with a complex

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self energy to account for its pion decay width and thus, implicitely, for the pion channels. The calculation showed that even at energies near the pion threshold, $E_\gamma \approx 150$ MeV, the contribution of the Δ-resonance is crucial. Even though a good agreement with the data was obtained, one aspect missing in the calculation is that unitarity is obeyed only approximately.

From calculations on pion photoproduction (see, e.g. [7,8]) it is known that the unitarity constraint, which can for example be imposed via Watson's theorem connecting (π, π) and (γ, π) amplitudes, is crucial to obtain the correct interference between the resonant and the background contributions. In this work we have improved on the calculation of Ref. [6] in this respect and have arrived at a unitary description of pion scattering, pion photoproduction and compton scattering on the nucleon.

2. Outline of the model

A simple approach that is particularly suited for imposing the unitarity constraint while keeping at the same time a direct link to the basic Feynman diagrams, is the K-matrix approach. In this approach the T-matrix, $S = 1 + 2iT$, is represented as $T = K/(1 - iK)$, from which it is evident that the scattering matrix S is unitary when K is hermitian.

The K-matrix formalism results from the Bethe-Salpeter equation in the approximation that the principal value of the loop integrals is neglected and only the contribution from the discontinuity is kept. Stated differently, the particles forming loops are taken to be on the mass shell. The role of the principle-value part has been studied in the framework of various quasipotential reductions of the Bethe-Salpeter equation in refs. [9,10] and [11,12]. Both groups use rather soft cutoff formfactors, and therefore the effect of the principle-value integral is not very large. It can moreover be argued (see e.g. Ref. [13]) that this effect will be mainly a renormalization of the coupling constants and the baryon masses. In our calculations these are taken to be the physical values where known and are otherwise treated as free parameters; we believe that such renormalizations are of small importance in the energy regime we consider. In addition the K-matrix prescription is simple.

We have employed the K-matrix approach in the space $\pi N \oplus \gamma N$. In this way Compton scattering is investigated together with the pion-nucleon scattering and pion photoproduction. Only in this way the unitarity constraint can be satisfied within the model space, however the two (and more) pion channels, which are known to become important at energies in excess of 400 MeV, are not included. The additional advantage is that all three processes are calculated *consistently* which puts stronger constraints on the model parameters.

Due to time-reversal invariance the partial wave K-matrix is a real and symmetric 4×4 matrix in a basis spanned by two pion-nucleon channels, corresponding to different values for the total πN isospin (1/2 and 3/2) and two photon-nucleon channels, corresponding to different helicities (or, equivalently, electric and magnetic radiation). For the partial wave decomposition we use the Jacob–Wick formalism as given in appendices of Refs. [4,14].

The K-matrix is approximated by the sum of tree-level diagrams including direct (s-type) and crossed (u-type) 'driving' terms with intermediate nucleon, N^* (Roper)- and Δ-resonances with *real* self energies equal to the mass of the resonances. The one pion-(one photon-) nucleon decay width of the resonances, as well as vertex corrections, are generated dynamically when calculating the T-matrix.

In the t-channel we include:
- for pion scattering, σ- and ρ-meson exchanges, where the σ-meson simulates a coherent 2π-exchange;
- for pion photoproduction, the π-, ρ- and ω-meson, where the latter two have only a marginal effect on the calculated quantities and the first is necessary to ensure current conservation;
- for Compton scattering, the π^0-meson exchange with the destructive interference.

In addition, the Kroll-Rudermann contact term is included in pion photoproduction to restore gauge invariance.

The Lagrangian we use is a standard one, see e.g. Ref. [13] for the strong-interaction and Refs. [6,15] for the e.m. terms. In the πNN coupling vertex we have allowed for a mixture of pseudo-scalar and pseudo-vector coupling specified by parameter χ. Note that according to [13] this parameter enters in the strength of the pseudo-scalar (pseudo-vector) coupling as $\chi/(1+\chi)$, respectively $1/(1+\chi)$. The

Δ-state is included via the spin-$\frac{3}{2}$ Rarita-Schwinger propagator, which off-shell contains also a spin-$\frac{1}{2}$ background. In each of the vertices involving the Δ therefore also an off-shell coupling parameter enters, which determines the coupling to the spin-$\frac{1}{2}$ sector of the Rarita-Schwinger propagator. These off-shell couplings appear to be of crucial importance to reproduce the data.

3. Results, discussion and conclusions

As emphasized, in the calculations the unitarity constraint is satisfied, even to higher orders in α, the fine structure constant. For πN scattering the higher order corrections in α are negligible and we will therefore discuss this case first to fix the pion coupling parameters.

Our model for πN scattering is very similar to that of Goudsmit et al. [13]. The main difference is that, since we are interested in somewhat higher energies, we have also included the Roper-resonance in the calculations. This improves the fit in the P_{11} channel as to be expected, but hardly influences any of the other partial-wave amplitudes.

To investigate the effect of the strong couplings we have used the results of three different fits to the πN phase shifts. Two of these, parameter sets # 1 and # 2 in Table 1, have been taken from Ref. [13]. These two parameter sets allow for the investigation of the effect of pseudo-scalar v.s. pseudo-vector πNN-coupling. To study the effect of the off-shell coupling in the πNΔ vertex (characterized by the parameter z_π) we have analysed also the third set given in Table 1.

It should be noted that for any change in a specific parameter the remaining ones are refitted to the π-N scattering data. As can be seen from Table 1 by comparing sets # 1 and # 2 this may introduce considerable changes in other parameters such as those for the σ and ρ meson exchanges. Eventhough this may obscure the dependence on any single parameter we have choosen to do so since our aim is to study the effects on compton scattering, keeping the agreement for π-N scattering as good as possible.

All three parameter sets given in Table 1 give a comparable overall fit to the Arndt partial-wave data [16]. Parameter set # 3 gives somewhat better results at higher energies in the S_{11}-channel but slightly worse results in the S_{31}-channel. Since for the first two parameter sets our results are very similar to those of Ref. [13] we will not discuss these any further in this short letter.

Table 1

Different sets of parameters used in the calculation of the pion-nucleon scattering. In the definition of the interaction Lagrangian Ref. [13] is followed, only the PV/PS mixing parameter x is renamed to χ. Parameter sets # 1 and # 2 correspond to two fits to the πN scattering data as presented in Ref. [13], at the extremes of their parameter spectrum. All parameters not explicitely mentioned are taken from this work with $g_{\pi\pi\rho} = 6.065$. The Roper resonance is included following Ref. [15] with $H = 0.145$ and vanishing width. Only its one-pion partial decay width is generated dynamically

set #	$g_{\pi NN}$	χ	$g_{\pi N\Delta}$	z_π	G_σ	$g_{\rho NN}$	κ_ρ
1	12.95	0.0	2.19	−.34	23	5.87	2.1
2	12.95	0.2	2.19	−.34	43	2.90	2.1
3	12.95	0.0	2.19	−.16	28	5.40	2.1

Table 2

Different sets of parameters for the $\gamma N\Delta$ vertex that give a comparable fit to the cross-section data. The parameters for the ω meson couplings are taken from Ref. [15]. For the electromagnetic decay of the Roper resonance we used [15] $G_p = -0.544$ and $G_n = +0.552$. The $E2/M1$ decay ratio is defined according to Eq. (1)

set #	G_1	z_1	G_2	z_2	$R(E2/M1)$
1	4.3	0.15	2.0	4.0	−4.47%
2	4.3	0.0	6.0	1.8	−0.57%

The results of our calculations for the pion photoproduction partial-wave amplitudes and the Compton-scattering cross section are compared with the data in Figs. 1 and 2. In these calculations only the four parameters of the γNΔ vertex have been optimized to reproduce the Compton scattering data [18,19]. As shown in Table 2, there is a range of values for G_2, compensated by an appropriate change in z_1 and z_2, for which a comparably good fit can be obtained (calculations A abd B in Fig. 2). A best fit to the data for Compton scattering is obtained with parameter set # 1 from Table 2.

A parameter often quoted for the NΔγ-vertex is the E2/M1 ratio for the electromagnetic decay of the Δ-resonance. This ratio is however not directly related to physical observables due to background contributions.

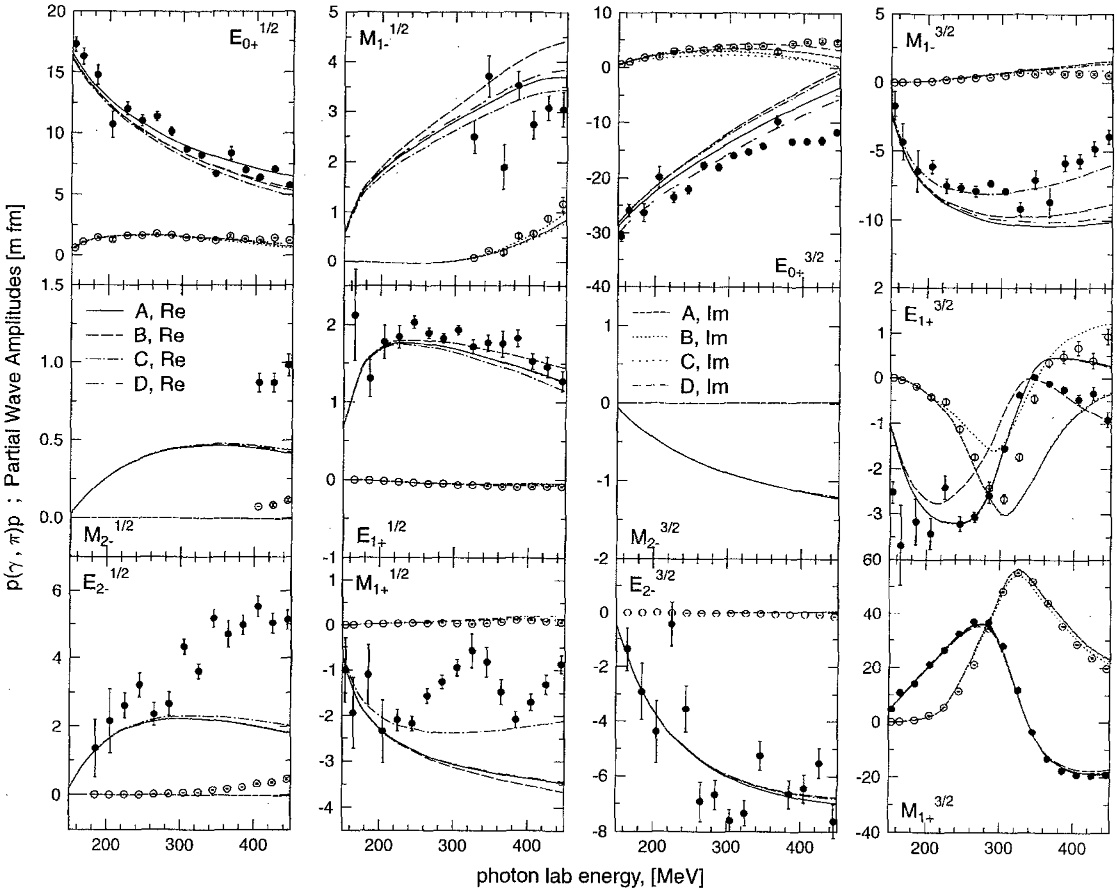

Fig. 1. Calculated pion-photoproduction partial-wave amplitudes are compared with the analysis of Ref. [17]. Calculation A (Table 1, set # 1 and Table 2, set # 1) gives the best fit to π-N scattering and compton scattering. In calculations B (Table 1, set # 1 and Table 2, set # 2), C (Table 1, set # 3 and Table 2, set # 1) and D (Table 1, set # 2 and Table 2, set # 1) the parameters have been changed to indicate the model dependence. The parameters have been choosen such as to maintain a reasonably good fit to π-N scattering.

The values given in Table 2 are defined as the ratio of the electric and magnetic decay rate of an 'on-shell' Δ-resonance. As such it depends only on the parameters G_1 and G_2 and *not* on the off-shell coupling parameters z_1 and z_2 [7],

$$R(E2/M1) = -\frac{2G_1 - G_2 M_\Delta/M}{2G_1(3M_\Delta + M)/(M_\Delta - M) - G_2 M_\Delta/M} \quad (1)$$

where M (M_Δ) is the nucleon (Δ-resonance) mass. The predictions for $R(E2/M1)$, as given in Table 2, thus vary strongly with G_2 while keeping agreement with the data where the variation of G_2 is compensated with a variation of the off-shell parameters. It should be noted that when a E2/M1 ratio is extracted from ratio's of partial wave amplitudes, more directly related to physical observables, the model dependence is reduced.

We find that in pion photoproduction the partial wave amplitudes are largely insensitive to the off-shell parameters. For example, from Fig. 1 one can see that the different choices for the $\gamma N\Delta$ parameters in Table 2 affect only $E_{1+}^{3/2}$ multipole amplitude [3] at energies

[3] The notation $\mathcal{M}_{L\pm}^{I}$ is used in pion photoproduction where $\mathcal{M}$ stands for the electric ($\mathcal{M} = E$) or magnetic ($\mathcal{M} = M$) type of the photon with the final πN state characterized by the orbital angular momentum L, parity $p = (-1)^{L+1}$, total angular

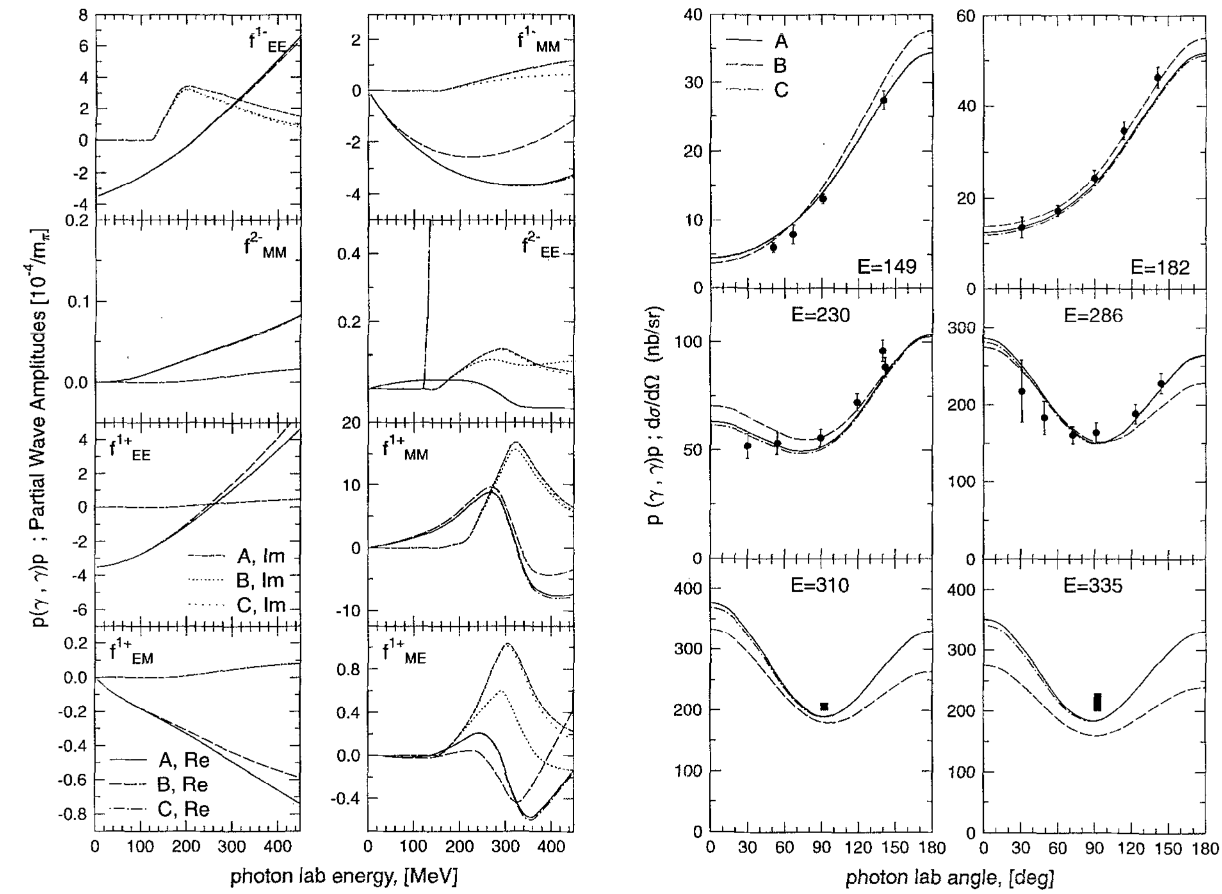

Fig. 2. A comparison of different calculations for Compton scattering where the same parameters are used as in Fig. 1. The data for the Compton-scattering cross section are from Ref. [18,19].

exceeding 300 MeV. None of the other partial wave amplitudes are affected.

Compton-scattering, in contrary, clearly distinguishes the parameter sets of Table 2, see curves A and B in Fig. 2. Note that mainly the f^{2-}_{EE} and f^{1+}_{ME} amplitudes are affected, which are related to the $E^{3/2}_{1+}$ amplitude in the pion photoproduction channel and the f^{1-}_{MM} compton multipole. The $\gamma N\Delta$ vertex enters quadratically in the Compton amplitude quadratically while the strong vertices do no enter explicitely. Compton scattering is thus strongly dependent on the γNΔ parameters.

None of the different choices for the parameters for πN scattering given in Table 1 do noticeably affect the results for Compton scattering. Including a pseudoscalar coupling improves the agreement with the π-N-scattering data at higher energies but has hardly any influence on the pion photoproduction channel. There is a considerable sensitivity to the off-shell coupling parameter z_π in the πNΔvertex. As mentioned, the fit for the S_{31} πN phase shift is somewhat worse for parameter set # 3 in Table 1, but the agreement for pion photoproduction is considerably improved, especially for the $M^{1/2}_{1+}$ channel. Again, the Compton scattering calculations are not affected.

We find that in the calculation of the phases of the

momentum $J = L \pm 1/2$ and total isospin $I = 1/2, 3/2$. In pion scattering $L_{2I\,2J}$ is used. In Compton scattering the notation is $f^{L\pm}_{\mathcal{M}\,\mathcal{M}'}$ where the total angular momentum is $J = L \pm 1/2$ and parity is $p = (-1)^L$ for $\mathcal{M} = E$ or $p = (-1)^{L+1}$ for $\mathcal{M} = M$.

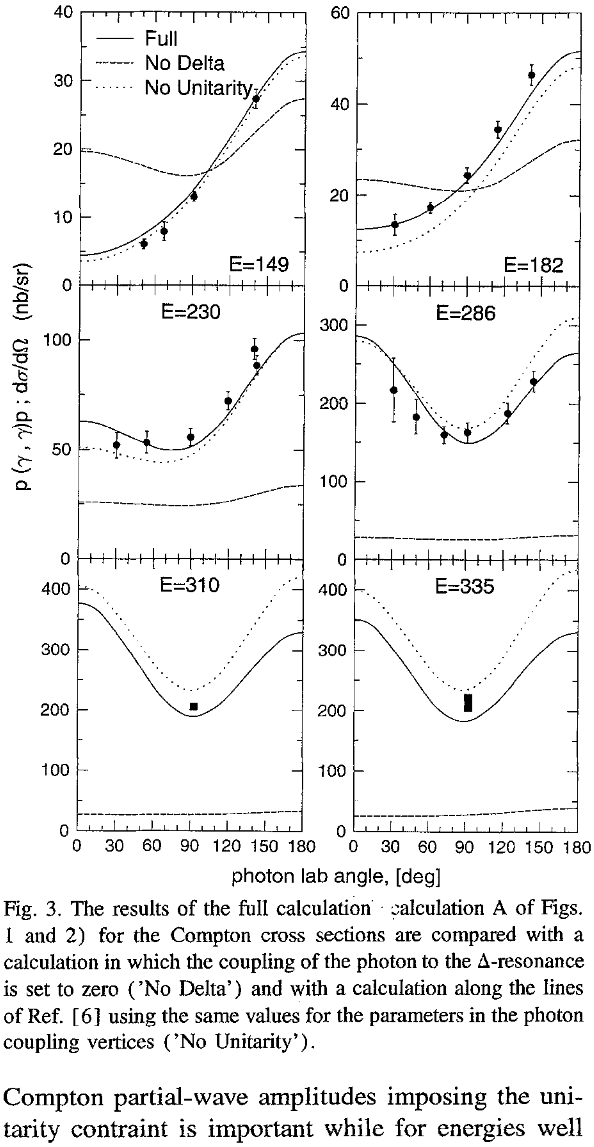

Fig. 3. The results of the full calculation (calculation A of Figs. 1 and 2) for the Compton cross sections are compared with a calculation in which the coupling of the photon to the Δ-resonance is set to zero ('No Delta') and with a calculation along the lines of Ref. [6] using the same values for the parameters in the photon coupling vertices ('No Unitarity').

Compton partial-wave amplitudes imposing the unitarity contraint is important while for energies well below the Δ-resonance the cross section in not much affected. This is shown in Fig. 3 where the results of the present calculations are compared with the non-unitarized tree approximation where a finite width for the Δ-resonance is included [6]. Only at energies where the Δ-resonance is dominating the spectrum and beyond, the differences are appreciable.

In Fig. 3 also the result of a calculation is shown where the photon-decay of the Δ is switched off. This demonstrates clearly the importance of the Δ even at energies near the pion threshold. Only due to a destructive interference at small angles and a strong constructive interference at large angles of the nucleon- and the Δ-amplitudes the characteristic vanishing of the cross section at 0 degrees can be accounted for.

In conclusion, Compton scattering is demonstrated to be most suitable for determining the parameters of the $\gamma N\Delta$ vertex since the Compton amplitude is largely *insensitive* to the strong channels, while it is very *sensitive* to the photon-Δ coupling. Quite the opposite is found for pion photoproduction. We have shown that in a rigorously unitary calculation of Compton scattering from the proton the interference of the nucleon and Δ-isobar amplitudes is crucial to understand the observed structure in the cross section as was also found in a tree level calculation [6]. The unitary constraint turns out to be important in the calculation of the cross section at photon energies exceeding 250 MeV.

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References

[1] M. Gell-Mann, M.L. Goldberger and W.E. Thirring, Phys. Rev. 95 (1954) 1612 .
[2] A.M. Baldin, Nucl. Phys. 18 (1960) 310;
D.M. Akhmedov and L.V. Fil'kov, Nucl. Phys. B 125 (1977) 530;
I. Guiasu, C. Pomponiu and E.E. Radescu, Ann. of Phys. 114 (1978) 296;
A.I. L'vov, V.A. Petrun'kin and S.A. Startsev, Yad. Fiz. 29 (1979) 1265, [Sov. J. Nucl. Phys. 29 (1979) 651].
[3] A.C. Hearn and E. Leader, Phys. Rev. 126 (1962) 789 .
[4] W. Pfeil, H. Rollnik and S. Stankowski, Nucl. Phys. B 73 (1974) 166.
[5] I. Caprini, I. Guiasu and E.E. Radescu, Phys. Rev. D 25 (1982) 1808 .
[6] V. Pascalutsa and O. Scholten, Nucl. Phys. A 591 (1995) 658.
[7] R.M. Davidson, N.C. Mukhopadhyay and R.S. Wittman, Phys. Rev. D 43 (1991) 71 .
[8] M. Vanderhaegen, K. Heyde, J. Ryckebusch and M. Waroquier, Nucl. Phys. A 595 (1995) 219.

[9] B.C. Pearce and B.K. Jennings, Nucl. Phys. A 528 (1991) 655.

[10] T.-S.H. Lee and B.C. Pearce, Nucl. Phys. A 530 (1991) 532.

[11] F. Gross and Y. Surya, Phys. Rev. C 47 (1993) 703 .

[12] Y. Surya and F. Gross, Phys. Rev. C 53 (1996) 2422 .

[13] P.F.A. Goudsmit, H.J. Leisi, E. Matsinos, B.L. Birbrair and A.B. Gridnev, Nucl. Phys. A 575 (1994) 673.

[14] F.A. Berends, A. Donnachie and D.L. Weaver, Nucl. Phys. B 4 (1967) 1.

[15] H. Garcilazo and E. Moya de Guerra, Nucl. Phys. A 562 (1993) 521.

[16] R.A. Arndt, I.I. Strakovsky and R.L. Workman, Phys. Rev. C 52 (1995) 2120 .

[17] R.A. Arndt, R.L. Workman, Z. Li and L.D. Roper, Phys. Rev. C 42 (1990) 1853 .

[18] E.L. Hallin et al., Phys. Rev. C 48 (1993) 1497 .

[19] G. Blanpied et al., Phys. Rev. Lett. 76 (1996) 1023.